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COMMENT

Spherically symmetric collapse and the naked singularity

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Abstract. It is shown that spherically symmetric collapse of a dust cloud in some cases may lead to a naked singularity.

It is well known that spherically symmetric gravitational collapse leads to a singularity of infinite density once the boundary of the system goes below the Schwarzschild radius ($r = 2m$). The Schwarzschild surface or the event horizon for the vacuum exterior metric has the property that future time-like or null trajectories pass through it only towards the interior region. In other words the system leads to a black hole.

Yodzis *et al* (1973) have recently pointed out that under certain conditions even spherically symmetric collapse may lead to a naked singularity—the singularity which can be observed by an external Schwarzschild observer. Such a singularity was, however, noted much earlier by Banerjee (1967) in the case of inhomogeneous spherically symmetric collapse of a dust cloud.

The line element and the solutions for such a system were given in a special case (Landau and Lifshitz 1962) as

$$ds^2 = dt^2 - r^2(d\theta^2 + \sin^2\theta \, d\phi^2) - e^{\omega} \, dR^2 \tag{1}$$

where

$$e^{\omega} = (r')^2 \tag{2}$$

and

$$(t_0(R) - t) = \tfrac{2}{3} r^{3/2} F^{-1/2}, \tag{3}$$

F being an arbitrary function of the radial coordinate R alone. Also

$$\dot{r}^2 = F/r \tag{4}$$

and

$$8\pi\rho = \frac{F'}{r' r^2}. \tag{5}$$

ρ being the rest mass density.

It was pointed out that the singularity $g_{11} = r' = 0$, where the radial separation locally vanishes although the circumferential distance still remains finite and density increases indefinitely, will appear before the collapse $r \to 0$ noted by Landau and Lifshitz only when $t_0'(R)$ is negative in the region under consideration.

Now one can proceed further to find also the conditions under which such a singularity would appear even before the boundary goes beyond the horizon.

Let $t = t_s$ be the time when the singularity $g_{11} = r' = 0$ appears at the boundary of the dust cloud, $t = t_h$ being the instant that the horizon crosses the surface. In view of (4) and (5) F and F' are everywhere finite within the system. From (2) t_0 is a finite positive function of R for a collapse and t_0' is also finite throughout the system. Now from (3) at the boundary of the dust cloud ($R = R_b$)

$$t_h = t_0(R_b) - \frac{4m}{3} \tag{6}$$

where $F(R_b) = 2m$, m being the gravitational mass of the sphere as is seen from the conditions of fit at the boundary. Again from the equation (7) of Banerjee (1967) at $R = R_b$

$$\begin{aligned} t_s(R_b) &= t_0(R_b) + 2\left(\frac{F}{F'}t_0'\right)_{R=R_b} \\ &= t_0(R_b) + 4m\left(\frac{t_0'}{F'}\right)_{R=R_b}. \end{aligned} \tag{7}$$

Now, in order that initially ($t = 0$) the boundary of the system has a radius greater than the Schwarzschild radius and also that the singularity appears at the boundary before it collapses below the horizon the following conditions are to be satisfied:

$$0 < t_s < t_h. \tag{8}$$

The inequality (8) is equivalent to

$$-\frac{3}{2}\frac{t_0(R_b)}{2m} < 3\left(\frac{t_0'}{F'}\right)_{R=R_b} < -1. \tag{9}$$

It is clear from (9) that $t_0'(R_b)$ has to be negative and the singularity in such a case is naked.

References

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